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THE GRADED NUMBERS
IN THE ANALYTIC HIERARCHY PROCESS

In the paper, the Analytic Hierarchy Process with labels expressed in the form of graded numbers is considered. It is shown that this approach is more general than the well known Saaty approach and that the use of graded numbers has some advantage over the possible use of fuzzy numbers. An order for graded numbers is also proposed.

1. Introduction

In many situations, we use measures or quantities which are not exact but approximate. In those cases, the concept of fuzzy number is more adequate than that of real number. Based upon the usual operations with real numbers, Zadeh's extension principle gives rise to arithmetic with fuzzy numbers, which is commonly used.

The Analytic Hierarchy Process (AHP) proposed by Saaty [12] is a very popular approach of multicriteria decision making (MCDM) that involves qualitative data and has been applied for the last twenty years in many situations of decision making.

In the literature, it can be observed that many authors treat the AHP from the point of view of fuzzy numbers, but we think that the operations are only approximate because it is not certain that the composition of two fuzzy numbers is a new fuzzy number.

In this paper, we consider the tools which are needed to solve the Analytic Hierarchy Process (AHP) in which the preferences are expressed by means of linguistic labels, whose values are stated as Zadeh's graded numbers. These numbers are analogous to the families of α-cuts of Zadeh's fuzzy numbers. More specifically, each Zadeh's graded number is defined as a non-increasing family of real bounded intervals, indexed by the unit interval [4]–[6].

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This difference appears advantageous for the graded numbers, whose arithmetic is a simple extension of the Interval Analysis [8], [9]; while the fuzzy numbers need additional conditions in order to be operated via their α-cuts. In this way and under appropriate conditions, the properties of the fuzzy numbers can be transferred to the graded numbers and vice-versa [1], [10].

In order to apply these methods, we need to operate and rank the graded numbers associated with each preference. However, as we have already said for the fuzzy numbers, the arithmetic results as a natural extension of the operations with real numbers, while the situation is different for the ranking. In addition, the order for the rewards implies an order for their corresponding alternatives and we must finally choose "the best" alternative.

The paper is divided into six sections. Firstly, we present the general approach of Saaty and the principal estimators of the weights. In the section 3, we briefly review the fundamental aspects concerning Zadeh's graded numbers, considering only those operations which are needed in order to solve Saaty's decision problem considered earlier. In the following section we consider certain orders for real intervals and for graded numbers. In section 5, we present a general problem with graded number, the Saaty approach being a particular case. Finally, we end with the conclusions.

2. The Analytic Hierarchy Process

The analytic hierarchy process (AHP) developed by Saaty is a widely used multicriteria decision method for multiple alternative decisions. This method was invented for subjective evaluation of alternatives that are organized in a hierarchical structure. At the top level, the criteria are evaluated and at the lower levels, the alternatives are evaluated by each criterion. The decision maker explains his/her preferences separately for each level.

It is necessary to evaluate individual alternatives, deriving weights for the criteria, construct the overall rating of the alternatives and identify the best alternative.

The AHP uses hierarchic or network structures to represent a decision problem. At each level of the hierarchy pairwise comparisons of decision elements (either criteria or alternatives) are used to arrive at priority scores of the elements under consideration.

Suppose an expert elicitation of intensity of preference of element A_i over element A_j is a_{ij}. For quantifying these values a_{ij} Saaty gives the following scale.

Let us denote the alternatives by $\{A_1, A_2, ..., A_n\}$ (n is the number of alternatives being compared), their actual weights by $\{w_1, w_2, ..., w_n\}$ and the matrix of the ratios of all weights by $\mathbf{W} = [\, w_i / w_j \,]$. The matrix of pairwise comparisons $\mathbf{A} = [a_{ij}]$ represents the intensities of the expert's preference between individual pairs of alternatives

Table 1

Values for Saaty's Analytic Hierarchy Process

Definition	Scale	Explanation
A_i and A_j are equally important	1	Two elements contributed equally to the property.
A_i is moderately more important than A_j	3	Experience and judgment slightly favour one element over another.
A_i is strongly more important than A_j	5	Experience and judgment strongly favour one element over another.
A_i is very strongly more important than A_j	7	An element is strongly favoured and its dominance is demonstrated in practice.
A_i is extremely more important than A_j	9	The evidence favouring one element over the other is of the highest possible order of affirmation.
The scales 2, 4, 6 and 8 are also used and represent compromises among the tabulated scales.	2, 4, 6, 8	Compromise is needed between two adjacent judgments.

(A_i versus A_j, for all $i, j = 1, 2, ..., n$) chosen usually from a given scale. When there are n alternatives $\{A_1, A_2, ..., A_n\}$ a decision maker compares a pair of alternatives for all possible pairs, $n(n-1)/2$, then a comparison matrix **A** is obtained, where the element a_{ij} shows the preference weight of A_i obtained by comparison with A_j.

$$\mathbf{A} = \begin{pmatrix} 1 & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ 1/a_{12} & 1 & \dots & a_{2j} & \dots & a_{2n} \\ . & & & & & \\ 1/a_{1j} & 1/a_{2j} & \dots & a_{ij} & \dots & a_{in} \\ . & & & & & \\ 1/a_{1n} & 1/a_{2n} & \dots & 1/a_{in} & \dots & 1 \end{pmatrix}$$

The elements a_{ij} are considered to be estimates of the ratios w_i/w_j where $w = \{x_1, x_2, ..., x_n\}$ is the vector of actual weights of the alternative, determined such that the matrix **W** is a close approximation to matrix **A** according to some metric which is what we want to find.

The following methods can be used for finding the vector of weights w:

• In the eigenvector method [12] the vector of weights is an eigenvector corresponding to the maximum eigenvalue $\lambda_{\max}$ of the matrix **A**. According to the Perron–Frobenius Theorem, the eigenvalue $\lambda_{\max}$ is positive and real. Furthermore, the vector w can be chosen with all positive coordinates. It is a normalized solution of the following equation:

$$\mathbf{A}w = \lambda_{\max} w$$

• The geometric means method (also known as the logarithmic least-squares method (LLSM)) where the approximating vector w has elements of the form

$$w_i = \left(\prod_{i=1}^{n} a_{ij} \right)^{1/n}, \quad i = 1, 2, ..., n.$$

The vector w is usually normalized so that the sum of the elements is one.

This means that an estimate v_i of the priority of A_i is found by taking the geometric mean of the a_{ij} over all $j = 1, 2, ..., n$. These v_i's can be normalized, however, for the purpose of establishing a statistically significant rank order of alternatives the normalization is not necessary.

• A simple additive weighting method (SAWM) is used to calculate the priorities for each alternative across all criteria.

$$w_i = \frac{\sum_{j=1}^{n} a_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}}.$$

3. Some basic aspects on graded numbers in relation with fuzzy number

Fuzzy decision problems require the processing of fuzzy variables and numbers that represent vague or imprecise information. Many methods have been introduced in the literature to compute information on fuzzy variables. Based on Zadeh's extension principle [14], several authors suggest several algorithms to complete the fuzzy weighted average.

On the other hand, it is well known that the multiplication and other related operation of two fuzzy numbers is not a new fuzzy number [7]. So, if we consider the fuzzy number related to fig. 1, for the increasing part of fuzzy number and $w \in [0, 1]$,

$$T_1 \circ T_2 = [A_1 + \alpha(M_1 - A_1)].[A_2 + \alpha(M_2 - A_2)] =$$

$$A_1.A_2 + \alpha[A_1(M_2 - A_2) + A_2(M_1 - A_1)] + \alpha^2 A_2(M_1 - A_1)(M_2 - A_2)$$

it is obvious that the product of two triangular fuzzy numbers is not a triangular fuzzy number, but one verifies this result for graded number.

Zadeh's graded numbers are defined as non-increasing families of bounded real intervals, indexed by the unit interval [0, 1]. Its operations are defined as a natural extension of the interval operations. So, analogous results to those which are commonly used for the families of α-cuts of fuzzy numbers, are easily obtained for the graded numbers.

A detailed study of such definitions and results can be found in the references cited [4], [5]. Here, we only consider the definition of Zadeh's graded number (using compact intervals) and the operations which are used to solve an AHP decision problem.

A Zadeh's graded number is defined as any mapping $\psi : [0,1] \to \{[A,B] : A \leq B \in R\}$, which assigns to each $\alpha \in [0,1]$ the interval $[\, a(\alpha), b(\alpha)\,]$ such that $\forall \alpha, \beta \in [0,1]$, $[\, \alpha < \beta \Rightarrow a(\alpha) \leq a(\beta) \leq b(\beta) \leq b(\alpha)\,]$.

Hereafter, we shall only use this kind of numbers, which will be called graded numbers for brevity.

Obviously, each graded number is determined by two functions $a,b : [0,1] \to R$, which satisfy the following three conditions:

- the function $a(\alpha)$ is non-decreasing,
- the function $b(\alpha)$ is non-increasing, and
- $a(1) \leq b(1)$.

The set of graded numbers is denoted by $G_Z(R)$. Obviously, this set extends the real line *R*, because each real number *P* can be identified with the graded number given by the constant functions $a(\alpha) = b(\alpha) = P,\ \forall \alpha \in [0,1]$.

For convenience, we use in this paper one specific kind of graded numbers: the "triangular graded number". To be more precise, we have:

Definition 3.1. We give the terms:

- *Triangular graded number* to any graded number determined by two linear functions $a,b : [0,1] \to R$, whose graphs describe a triangle. More precisely, for any three real numbers $A \leq M \leq B$, we have the triangular graded number $\tau_{(A,M,B)}$ = $[\, a(\alpha), b(\alpha)\,]$ determined by the following functions:

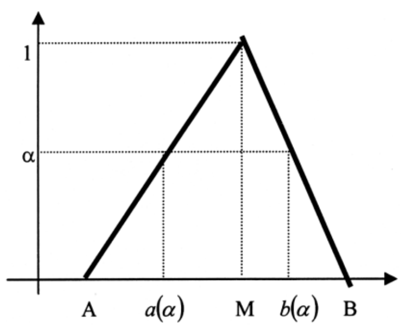

Fig. 1. Triangular graded number

$$a(\alpha) = A + \alpha(M - A) \quad b(\alpha) = B - \alpha(B - M)$$

Let us consider the label that correspond with the expression "A_i is moderately more important than A_j" with associated value 3. The corresponding triangular graded number, if the label is symmetric with amplitude 1, is the following:

$$a(\alpha) = 2 + \alpha \quad b(\alpha) = 4 - \alpha$$

then

$$\tau_{(2,3,4)} = [2+\alpha, 4-\alpha]$$

In particular, we have:

$$\begin{cases} \alpha = 0 \rightarrow [2, 4] \\ \alpha = 1 \rightarrow [3, 3]. \end{cases}$$

Operations. For any positive graded numbers $\psi(\alpha) = [a(\alpha), b(\alpha)]$, $\psi_1(\alpha) = [a_1(\alpha), b_1(\alpha)]$, $\psi_2(\alpha) = [a_2(\alpha), b_2(\alpha)]$ and any real number P (in our case $P > 0$), we are interested here in the operations related with AHP problems, which are defined as follows:

a) *Product by a scalar*:

$$(P\psi)(\alpha) := \{Px : x \in \psi(\alpha)\} = \begin{cases} [Pa(\alpha), Pb(\alpha)], \text{ if } P \geq 0 \\ [Pb(\alpha), Pa(\alpha)], \text{ if } P \leq 0 \end{cases}$$

b) *Addition*:

$$(\psi_1 + \psi_2)(\alpha) = \{x + y : x \in \psi_1(\alpha), y \in \psi_2(\alpha)\} = [a_1(\alpha) + a_2(\alpha), b_1(\alpha) + b_2(\alpha)]$$

c) *Division*:

$$(\psi_1 / \psi_2)(\alpha) = \{x/y : x \in \psi_1(\alpha), y \in \psi_2(\alpha)\} = [a_1(\alpha)/b_2(\alpha), b_1(\alpha)/a_2(\alpha)]$$

4. Ranking of Graded Intervals

In this section, we begin considering several orders in the set $C = \{[A, B]: A \leq B \in R\}$ of real compact intervals. Afterwards, we extend those orders to a subset of $G_z(R)$.

According to the subject of our study, we consider that each interval represents an imprecise determination of a label. We also consider the degree *of optimism*, which is a subjective value given by the parameter $\lambda \in [0, 1]$. Associated with each value λ and each interval $I = [A, B]$, we have the number $M_{(\lambda)} = \lambda B + (1-\lambda)A$ (a precise but subjective estimation of the label). A pessimistic (resp. optimistic) decision-maker acts in accordance with the value $\lambda = 0$ (resp. $\lambda = 1$). First, he or she ranks the alternatives looking at the value $M_{(0)}(I) = A$ (resp. $M_{(1)}(I) = B$).

In the case of two alternatives with the same value $M_{(\lambda)}(I_i)$, hey are ranked according to the other extreme: the interval with the greatest B (resp. with the greatest A) is preferred.

Let us note that this is a lexicographic order; however, the second part is equivalent to the following: the interval with the greatest amplitude $B - A$ (resp. with the lowest amplitude $B - A$) is preferred.

Definition 4.1. Let us consider the parameter $\lambda \in [0, 1]$, called *degree of optimism*. Associated with each value of λ we define the following two total orders in the set C (where we consider arbitrary elements $I_i = [A_i, B_i]$; $i = 1, 2$ and the corresponding values $M_{(\lambda)}(I_i) = \lambda B_i + (1-\lambda)A_i$:

1. $I_1 \leq_{\lambda r} I_2 \Leftrightarrow M_{(\lambda)}(I_1) < M_{(\lambda)}(I_2)$ or $[M_\lambda(I_1) = M_\lambda(I_2)$ and $B_2 - A_2 \geq B_1 - A_1]$.
2. $I_1 \leq_{\lambda s} I_2 \Leftrightarrow M_{(\lambda)}(I_1) < M_{(\lambda)}(I_2)$ or $[M_\lambda(I_1) = M_\lambda(I_2)$ and $B_2 - A_2 \leq B_1 - A_1]$.

The reflexive, anti-symmetric, transitive and convex properties (which guarantee that $\leq_{\lambda\tau}$ and $\leq_{\lambda\sigma}$ are indeed total orders in C) are easily checked. We can represent each $[x, y] \in C$ by the point (x, y) belonging to the half-plane $y \geq x$, thus visualizing the orders defined above.

- For $\lambda = 0$ we first rank vertical half-lines (preferring the right lines to the left ones) and second, inside each half-line, we rank the points (preferring the high points to the low ones).
- For $\lambda = 1$ we first rank horizontal half-lines (preferring the high lines to the low ones) and second, inside each half-line, we rank the points (preferring the right points to the left ones).
- For $0 << \lambda << 1$, we first rank the parallel half-lines $(1-\lambda)x + \lambda y = m$ (according to the values of m) and second, we rank the points inside each half-line (which corresponds to different intervals with the same value of M_λ).

In order to apply Definition 4.1, in the case of graded numbers, we first summarize each graded number $\psi(\alpha) = [a(\alpha), b(\alpha)]$ into an interval, using the mean values obtained integrating the monotonic functions $a(\alpha)$ and $b(\alpha)$:

Definition 4.2. We consider the mapping $G_Z(R) \to C$, which maps each graded number $\psi(\alpha)=[a(\alpha),b(\alpha)]$ into the interval $[A(\psi),B(\psi)]$ defined as:

$$A(\psi):=\int_0^1 a(\alpha)\,d\alpha \qquad B(\psi):=\int_0^1 b(\alpha)\,d\alpha$$

In addition, for each $\lambda \in [0, 1]$, we consider the value

$$M_\lambda(\psi):=\ \lambda B(\psi)+(1-\lambda)A(\psi).$$

Firstly, we can partially rank the graded numbers, ranking its associated values $M_\lambda(\psi)$. This would be analogous to a method given by Gonzalez to rank fuzzy numbers [9, 17], which generalizes Yager's method [38] (corresponding to the case $\lambda = 1/2$). Let us remark that Gonzalez uses the Stieltjes integral, with respect to any additive measure defined on [0, 1].

Secondly, we must rank the different graded numbers with the same $M_\lambda(\psi)$. If they have associated different intervals [A, B], then we rank these intervals using Definition 4.1.

We are interested in extending this partial order to a total order. Thus, we must rank the different graded numbers with the same $M_\lambda(\psi)$. If they have associated different intervals [A, B], then we rank these intervals using Definition 4.1.

On the contrary, let us restrict ourselves to simple subsets of graded numbers. For example, for the set of triangular graded numbers, we can apply the method suggested above. Indeed, if $\tau_{(A,M,B)} \neq \tau_{(A',M',B')}$ but $A = \{A + M\}/2 = \{A'+M'\}/2$ and $B = \{B + M\}/2 = \{B' + M'\}/2$, then $A \neq A'$, $M \neq M'$ and $B \neq B'$.

Therefore, we can consider the levels $\alpha_r = 0$ (when $\kappa = r$) and $\alpha_s = 1$ (when κ = s) in order to rank such triangular numbers.

5. An illustrative example. Hierarchical composition of priorities

(Proposed by Saaty [34]). The difference is in the scale, we suppose that the scale is a graded number; in this case, a triangular graded number.

Table 2

Values for Saaty's Analytic Hierarchy Process

Comparative judgment	Saaty's Scale	Triangular number	Graded number
A_i and A_j are equally important	1	[1, 1, 1]	[1, 1]
A_i is moderately more important than A_j	3	[2, 3, 4]	[2 + α, 4 – α]

A_i is strongly more important than A_j	5	[4, 5, 6]	[4 + α, 6 – α]
A_i is very strongly more important than A_j	7	[6, 7, 8]	[6 + α, 8 – α]
A_i is extremely more important than A_j	9	[8, 9, 9]	[8 + α, 9]

The scales 2, 4, 6 and 8 are also used and represent compromises among the tabulated scale and calculated in the same form.

School Selection Example. Three high schools, A, B, C, were analysed from the standpoint of the author's son [12] according to their desirability. Six independent characteristics were selected for the comparison: learning, friends, school life, vocational training, college preparation and music classes.

The pairwise judgment matrices were as shown in the following tables.

Table 3

Comparison of characteristics with $\alpha = 1/2$ and with respect to overall satisfaction with school

	Learning	Friends	School life	Vocational training	College preparation	Music classes	Weight interval
Learning	[1, 1]	[7/2, 9/2]	[5/2, 7/2]	[1, 1]	[5/2, 7/2]	[7/2, 9/2]	[.193, .301]
Friends	[9/40, 7/24]	[1, 1]	[13/2, 15/2]	[5/2, 7/2]	[11/60, 9/40]	[1, 1]	[.157, .226]
School life	[7/24, 5/12]	[15/112, 13/84]	[1, 1]	[11/60, 9/40]	[11/60, 9/40]	[1/7, 11/60]	[.027, .037]
Vocational training	[1, 1]	[7/24, 5/12	[9/2, 11/2]	[1, 1]	[1, 1]	[7/24, 5/12	[.111, .156]
College preparation	[7/24, 5/12]	[9/2, 11/2]	[9/2, 11/2]	[1, 1]	[1, 1]	[5/2, 7/2]	[.190, .283]
Music classes	[9/40, 7/24]	[1, 1]	[11/2, 13/2]	[5/2, 7/2]	[7/24, 5/12]	[1, 1]	[.145, .213]

In order to obtain the ranking for different schools it is necessary to make the following operations.

5.1. Obtaining weights

In the case of graded numbers, the anterior operation gives the result $[12 + 4\alpha, 20 - 4\alpha]/6$, which for $\alpha = 0 \Rightarrow [12, 20]$ and for $\alpha = 1 \Rightarrow [16, 16]$ corresponding to the extremes and the central values, respectively. For $\alpha \in (0, 1) \Rightarrow$ the corresponding values are (12, 20). The corresponding interval $[A, B]$ given in Definition 4.2 is, $[A, B] = [14, 18]$, corresponding with $\alpha = 1/2$.

Learning → [1, 1] + [7/2, 9/2] + [5/2, 7/2] + [1, 1] + [5/2, 7/2] + [7/2, 9/2] = [14, 18]

And so on for the rest of the criteria, being the sum for all criteria

SUM = [59.747, 72.680]

The final values in the anterior step are divided by the SUM. According to the expression given in Section 3, we obtain for the first value:

$$\frac{[14,\ 18]}{[59.747, 72.680]} \Rightarrow \left[\frac{14}{72.680}, \frac{18}{59.747}\right] = [0.193,\ 0.301]$$

The same procedure is applied to the rest of the values. The weights related to criteria are reflected in the last column of Table 3.

In the following we give the values/weights associated with the alternatives for each of the criteria and calculated in the same way.

Table 4

Comparison of schools

		A	B	C	SUM
Learning	A	[1,1]	[7/24, 5/12]	[5/12, 3/4]	[.121, .206]
	B	[5/2, 7/2]	[1,1]	[5/2, 7/2]	[.426, .762]
	C	[1,2,3]	[7/24, 5/12]	[1,1]	[.198, .373]
Friends	A	[1,1]	[1,1]	[1,1]	[.333, .333]
	B	[1,1]	[1,1]	[1,1]	[.333, .333]
	C	[1,1]	[1,1]	[1,1]	[.333, .333]
School life	A	[1,1]	[9/2, 11/2]	[1,1]	[.395, .522]
	B	[11/60, 9/40]	[1,1]	[11/60, 9/40]	[.083, .101]
	C	[1,1]	[9/2, 11/2]	[1,1]	[.395, .522]
Vocational training	A	[1,1]	[17/2, 9]	[13/2, 15/2]	[.627, .763]
	B	[1/9, 17/144]	[1,1]	[11/60, 9/40]	[.051, .058]
	C	[15/112, 13/84]	[9/2, 11/2]	[1,1]	[.221, .290]
College preparation	A	[1,1]	[5/12, 3/4]	[1,1]	[.210, .311]
	B	[3/2, 5/2]	[1,1]	[3/2, 5/2]	[.348, .679]
	C	[1,1]	[5/12, 3/4]	[1,1]	[.210, .311]
Music classes	A	[1,1]	[11/2, 13/2]	[7/2, 9/2]	[.542, .789]
	B	[11/60, 9/40]	[1,1]	[7/24, 5/12]	[.080, .198]
	C	[9/40, 7/24]	[5/2, 7/2]	[1,1]	[.202, .315]

5.2. Obtaining priorities

This step is related to obtaining the final ranking. For this purpose it is necessary to multiply the values from the column SUM in Table 4 by the corresponding values from the column Weight interval in Table 3 (with the operation given in section 3). The solution is given in Table 5.

Table 5

Final weights

	Intervals
A	[0.274, 0.532]
B	[0.220, 0.533]
C	[0.195, 0.407]

5.3. Comments

We obtain three interval values, the upper value is in accordance with the characteristic of optimism $\lambda = 1$, while the lower values represent the pessimism $\lambda = 0$, and the central value coincides with the degrees obtained in Saaty's method.

We see that the alternative *C* is dominated by the alternatives *A* and *B*, but the ranking between *A* and *B* is not so clear, because [0.220, 0.533] $\subset$ [0.274, 0.532].

For a degree of optimism $\lambda > 0.98$, the alternative *B* is preferred to *A* $(B \succ A)$, because $M_{(\lambda)}(A) < M_{(\lambda)}(B) \Rightarrow I_A \leq_{\lambda r} I_B$.

For $\lambda \leq 0.98$ is $A \succ B$, $M_{(\lambda)}(A) > M_{(\lambda)}(B) \Rightarrow I_B \leq_{\lambda s} I_A$.

For $\lambda = 0.98$ is $A \succ B$, $M_{(\lambda)}(A) = M_{(\lambda)}(B)$ and $B_2 - A_2 = 0.313 \leq 0.230 = B_1 - A_1$]. $\Rightarrow I_B \leq_{\lambda s} I_A$.

6. Conclusions

We present a new approach to the analytic hierarchy process in the general case in which the linguistic labels are known as graded numbers. We have demonstrated that the model is more general than the model proposed by Saaty.

Taking into account the definition of division given in section 3 it is natural that the final graded numbers show deviation towards the right, and since they are not symmetric for this reason the central value is not for $\lambda = 0.5$.

In the particular case in which we work with triangular numbers, the central value of the solution reproduces the solution of the original work of Saaty.

We exhibit the advantages for the graded numbers, whose arithmetic is a simple extension of the Interval Analysis; while the fuzzy numbers need additional conditions in order to be operated via their α-cut.

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Stopniowane liczby w analitycznym procesie hierarchicznym

Rozważa się analityczny proces hierarchiczny z etykietami w postaci liczb stopniowanych. Aby otrzymać ocenę najlepszej alternatywy albo uszeregowanie alternatyw, potrzebny jest całkowity porządek dla liczb stopniowanych występujących w problemie. Proponujemy definicję takiego porządku, opartą na dwóch subiektywnych aspektach: stopniu optymizmu/pesymizmu i upodobania do ryzyka/bezpieczeństwa. Ponieważ wiele operacji, np. iloczyn czy dzielenie, nie zachowuje trójkątności liczb rozmytych, w artykule stosuje się więc liczby stopniowane, analogiczne do liczb rozmytych. Operacje na nich są jednak prostym rozszerzeniem operacji na rzeczywistych przedziałach. Pokazano, że to podejście jest ogólniejsze od znanego podejścia Saaty'ego.